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# THE MATHEMATICS TEACHER

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VOLUME XII

MARCH, 1920

NUMBER 3

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ASSOCIATION OF MATHEMATICS TEACHERS OF  
NEW JERSEY. REPORT OF THE COMMITTEE OF  
FIRST-YEAR HIGH-SCHOOL MATHEMATICS.

(Concluded from last issue.)

## 8. Fractions.

(Note: Ability to factor at sight confidently and correctly is of great importance.)

(a) *Reduction to Lowest Terms.*—Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction. Many numerical examples.

Additional connection with arithmetical fractions is established by the numerical check, especially important in reduction of fractions. In checking, values must be substituted which do not give zero denominators.

(b) *Reduction to Common Denominator.*—Lowest common multiple explained, with reference to arithmetic. Abundant practice with monomial denominators would precede that with binomial and trinomial denominators, where factoring adds to the difficulties.

(c) *Addition and Subtraction.*—Postpone to intermediate algebra such examples as

$$x - a - \frac{3}{a^2 - x^2} - \frac{4a}{x^2}$$

with the confusing change of signs. Numerical checks may

help to avoid the troublesome error of entirely losing the common denominators.

(d) *Multiplication and Division of Fractions.*—This topic may precede (b) and (c).

Complex fractions should be omitted, except arithmetical ones when needed for checking the solution of an equation like

$$\frac{3x+1}{5x+1} = \frac{5}{7}$$

where  $x = \frac{1}{2}$ , or for examples in numerical substitution such as finding the values of  $(a+b)/(1-ab)$  when  $a = \frac{1}{2}$ ,  $b = \frac{1}{3}$ .

#### 9. *Fractional Equations and Problems. Literal Equations and Formulas.*

The process of clearing equations of fractions requires very thorough teaching. Equations with decimal coefficients; decimals and percentage; interest. The following formulas should be the basis of part of the work in literal equations, and in substitution:

- (1)  $br = p$  (percentage),
- (2)  $b + br = p$  and  $b - br = p$  (percentage of increase or decrease),
- (3)  $i = prt$  or  $i = pr \cdot \frac{n}{360}$  (interest),
- (4)  $i = pr \cdot \frac{n}{365}$  (accurate interest),
- (5)  $p + prt = a$  (amount).

Solve these formulas, for each of the letters in terms of the others.

(Note: T. Percy Nunn avoids the use of  $x$  to represent the unknown quantity until after a thorough study of formulas; the solution of simple equations is called by him and by other English authors "changing the subject of a formula.") After solving formulas concerning familiar subject matter such as mensuration for the various letters involved, let the student express each result as a verbal rule, and apply it to specific numerical examples.

Note: Omit special devices of method such as those often recommended for

$$\frac{6x+1}{15} - \frac{2x-4}{7x-16} = \frac{2x-1}{5}$$

and for

$$\frac{1}{x-1} - \frac{1}{x-2} = \frac{1}{x-3} - \frac{1}{x-4}$$

#### 10. *Ratio and Proportion.*

(a) *Ratio*.—Ratio is one of the most valuable topics in elementary mathematics. Emphasize the fact that a ratio, a fraction, and a division are all merely different ways of expressing the same thing, *i.e.*, three ways of indicating division. A decimal fraction or rate per cent. is only a camouflaged ratio. Practice in changing from one form of expression to the other, and in expressing ratio in all possible ways, *e.g.*, “Express as a common fraction and as a decimal to three significant figures the ratio of an inch to a foot, of a foot to a yard, of a yard to a rod, etc.”

(b) *Partition*.—Dividing a number in a given ratio. Mixtures, alloys, family budgets (*e.g.*, food, rent, clothes, miscellaneous, and savings proportional to 5, 21, 1, 1).

(c) *Proportion*.—(1) An equality of two ratios. The product of the means is equal to the product of the extremes (omit “alternation,” “inversion,” etc.). Numerical and literal equations in the form of proportions.

(2) Scale-drawing. Maps. (Express as a unit ratio the scale:  $\frac{1}{4}'' = 1'$ ,  $\frac{3}{4}'' = 1'$ , 3 in. = 1 mile, etc. The last answer is called the representative fraction of the map.) Actual house-plans, blue prints, and topographic or city maps should be used in the class-room to ascertain the scale used, measure and estimate distance, etc. The student may prepare a plan of the school or home grounds, an athletic field, a garage, or a floor-plan of a house or of the school room.

(3) Similar figures, protractor; angle measurement and construction. Height of tree by its shadow. Distance to inaccessible point by base-line, angle measurement, and scale drawing. (Examples in reducing and enlarging drawings or pictures, and the pantograph may be used.)

(d) *Inverse Ratio*.—The lever. “Work” problems.

## II. Graphs.

(a) Relative magnitudes from statistics shown by lengths of lines, sizes of rectangles.

(b) Circular graph.

Sectors and percentage. A good example would be one showing the make-up of the school in boys and girls of different courses of grades.

(c) Graph of a single linear equation in two unknowns.

(d) Simultaneous linear equations solved by graphs.

Graphs of inconsistent and dependent equations.

12. *Simultaneous Equations.*

(a) Elimination by addition or subtraction.

(b) Elimination by substitution.

Besides the usual check some examples may be verified by solving by the other method of elimination or by graphs. Sometimes the same examples may profitably be worked in the three ways.

(c) Literal equations.

(d) Three unknown quantities.

This is a topic of minor importance on which teachers are prone to linger in the interest of thoroughness. The work with three unknowns would best be sharply limited, say to three lessons.

(e) Problems.

Number system and digit puzzles. Sum of the angles of a triangle (the sum to be first derived by measurements with protractor and by paper-folding).

13. *Square Root.*

(a) Algebraic expressions.

Only integral expressions which are perfect squares of trinomials. Aim for a genuine understanding of the method, using perhaps  $a^2 + 2ab + b^2$  as a type example. Check by multiplication.

(b) Numerical square root.

Approximate roots to not over 4 significant figures. Finding the hypotenuse or arm of right angle triangle,  $c^2 = a^2 + b^2$ .

14. *Exponents.*

(a) Illustrate the laws of exponents without formal proof.

(b) Meaning of zero and negative exponents:

$a^2 \div a^2 = a^{2-2} = a^0$ , but  $a^2/a^2 = 1$ , Therefore  $a^0 = 1$ ;  
 $a^2 \div a^3 = a^{2-3} = a^{-1}$ , but  $a^2/a^3 = 1/a$ , Therefore  $a^{-1} = 1/a$ .

(c) Meaning of fractional exponents:

$a^{3/2} \times a^{3/2} = a^{3/2+3/2} = a^3$ , Therefore, because of the definition of square root,  $a^{3/2} = \sqrt{a^3}$ ; also  $(a^{1/3})^3 = a^{3/3} = a$ , Therefore,  $a^{1/3} = \sqrt[3]{a}$ .

Examples in exponents should deal with monomials only. They should include practice in writing very large numbers, such as those in astronomy, or very small number by using powers of 10, *e.g.*, three billion miles is  $3 \times 10^9$ ;  $2 \times 10^{-7} = ?$

15. *Radicals.*

Quadratic surds only, except under (a). All work with radicals closely associated with exponents. Fractional exponent and radical sign interchanged when helpful.

(a) Removal from the radical of a perfect power. Numerical square root by factoring method.

(b) Removal from the radical of a fraction.

(c) Four fundamental operations.

The laws understood.

$$2\sqrt{n} + 3\sqrt{n} = 5\sqrt{n},$$

$$\sqrt{m} \times \sqrt{n} = \sqrt{mn},$$

$$\frac{\sqrt{m}}{\sqrt{n}} = \sqrt{m/n},$$

$$(\sqrt{mn})^2 = mn.$$

(d) Rationalizing monomial denominators.

Application of (a) and (b), and (d) to sides of right triangles, altitudes of equilateral and isosceles triangles, and diagonals of rectangles. Find approximate value of fractions such  $\frac{\sqrt{5}}{\sqrt{3}}$  or  $\frac{\sqrt{2}-1}{\sqrt{2}}$  to four significant figures; do the work with and without rationalization, and make comparisons as to economy of figures and time, and as to degree of accuracy.

16. *Quadratics.*

## (a) Graphs.

Functions of  $x$  such as  $y = x^2$ ,  $y = x^3$ , etc. Solution of equations of the type of  $ax^2 + bx + c = 0$  by drawing the graph of  $y = ax^2 + bx + c$ .

Graphs help to make clear the nature of the roots, as well as the fact that two roots are possible.

## (b) Pure quadratics, with problems.

Find the radius of a circle given the area. Include practice in the evaluation of other formulas involving square root such as  $c^2 = a^2 + b^2$ ,  $s = \frac{1}{2}gt^2$ ,  $v = \pi R^2h$ , H.P.  $= ND^2/2.5$ .

(c) Solution of general quadratics by completing the square. Not more than two lessons should be taken, with very simple and carefully graded examples sufficient only to make the derivation of the formula intelligible.

## (d) Solution by formula.

This is the usual method, and one which is always available. Practice in obtaining numerical values of the roots to two places of decimals. Check by substitution, by graphs, or by completing the square. Checking such a root as  $2 - \sqrt{5}$  by substitution makes a good example in radicals. Omit difficult literal equations, such as those where either the constant term or one of the coefficients is a binomial.

## (e) Solution by factoring.

(f) Simultaneous equations, one of them linear, the other quadratic. Solution by substitution from the linear equation. Limit work to three lessons. Let each pupil solve four or five examples by graphs also, containing illustrations of the parabola, circle, hyperbola and ellipse: the equations used might be

$$\begin{array}{ll} (1) \begin{array}{l} y^2 = 4x, \\ x + y = 8; \end{array} & (2) \begin{array}{l} x^2 + y^2 = 25, \\ 3x + 4y = 25; \end{array} \\ (3) \begin{array}{l} xy = 12, \\ y = x + 4; \end{array} & (4) \begin{array}{l} x^2 + 4y^2 = 36, \\ x + y + 10 = 0. \end{array} \end{array}$$

## (g) Problems involving general quadratics.

New problems may also accompany each method of solution (c), (d), (e), and (f).

## X. GENERAL NOTES.

1. *Sequence of Topics*.—The syllabus is not intended to be rigid or restrictive concerning methods, nor concerning such matter as the order of topics. Many teachers, for example, will sometimes teach multiplication before subtraction. Topics will often overlap; preliminary work with a new topic may be carried on while completing the study of the preceding topic. Few recitations should be wholly without practice in verbal and algebraic expression or problems.

2. *Non-essential Topics*.—If necessary further to shorten the course, the following topics of little importance for first-year work could be omitted: algebraic square root, equations with three unknowns, simultaneous quadratics, type 6*b* of factoring all fractions with denominators other than monomials.

3. *Definitions and Rules*.—Few definitions or rules need be memorized, but there should be clear oral statements of methods and of explanations; too much should not be expected of beginners.

4. *Written Work*.—There is general agreement that written work of some sort should be handed in by the pupil daily, although the instructor's time will not permit him to grade all the papers carefully.

5. *Mental Algebra*.—There should be frequent practice in mental algebra and mental arithmetic.

6. *Reasonable Answers*.—A thoughtful preliminary guess or rough estimate of the answer and the habit of examining the reasonableness of the actual result will prevent many of the wildly absurd answers so often encountered.

7. *Significant Figures*.—Pupils should develop a certain amount of judgment as to how to carry approximate results. "To the nearest thousandth," "to three significant figures," etc., should be understood from the start. The technique of approximate multiplication and division, etc., probably should not be taught. It belongs in the shop or laboratory, where so much figuring of a certain kind may be required that the student will really appreciate the short methods.

8. *Initiative in Drill Exercises*.—The members of a class may occasionally be encouraged to make up or compose their own



individual examples for drill, *e.g.*, "Write down ten trinomials which are perfect squares, and factor each." "Multiply any quadratic trinomial in  $x$  by a binomial, checking by division." "If  $x=5$  fill in a second member for each of the following equations and solve:

$$3x-2=? \quad 5(x+3)-4(x-6)=? \quad (x+2)^2-(x+3)(x-1)=?$$

"Make up and solve five different sets of simultaneous equations with  $x=4$ ,  $y=3$ ."

#### XI. FIRST-YEAR MATHEMATICS WORD LIST.

Most of the words in the following list should be a part of the pupil's working vocabulary at the end of the first high-school year. Formal definitions need not be aimed at, but rather correct spelling, correct pronunciation and a knowledge of the meaning of the words as shown by their use in simple sentences.

Abbreviation, abscissa, absolute, abstract, accuracy, acre, acute, addend, addition, adjacent, adjoining, affected, aggregate, aggregation, algebra, algebraic, altitude, aliquot, amount, analysis, angle, angular, annulus, annuity, antecedent, applicable, approximate, arc, area, arithmetic, arrangement, ascending, associative, average, axiom, axis.

Balance, base, binomial, bisect, breadth.

Calculate, cancel, capacity, cardinal, census, center, centigrade, centimeter, checking, chord, circumscribe, circumference, classify, coefficient, collect, column, commission, common, commutative, compasses, complement, complete, complex, composite, composition, compound, concave, concentric, concrete, concise, condition, consecutive, consequent, consistent, constant, construction, contents, continuation, convenient, converge, convex, coordinates, correspondence, cross-section, curve, cylinder, cylindrical.

Data, decimal, decrease, deduce, deduction, definition, degree, denominate, denomination, denominator, depth, derive, descending, describe, determine, diagonal, diagram, diameter, difference, digit, dimensions, diminish, discount, dissimilar, distance, distributive, diverge, dividend, divisible, division, divisor.

Elementary, elevation, elimination, ellipse, equal, equation, equiangular, equilateral, equivalent, estimate, evolution, exactly, except, excess, exclude, explanation, exponent, expression, extreme.

Face, factor, Fahrenheit, focus, formula, fraction, function, fundamental.

Gauge, geometric, gram, graphic, gravity, grouping.

Height, hence, hexagon, horizontal, hundredth, hypotenuse.

Identical, identity, imaginary, inclination, incomplete, inconsistent, increase, independent, indeterminate, index, indicate, induction, inequality, innermost, inscribe, inspection, instructor, insurance, integer, integral, intercept, interest, interpretation, interior, inverse, involution, involving, irrational, isosceles, italics.

Kilogram.

Lateral, latitude, length, line, linear, liter, literal, longitude.

Manipulation, mathematics, maturity, maximum, mean, measurement, memorandum, meridian, meter, mill, millimeter, minimum, minuend, minus, miscellaneous, model, monomial, mortgage, multiple, multiplicand, multiplication, multiplier.

Necessary, negative, notation, number, numeration, numerator, numerical.

Oblique, obtuse, occurrence, octagon, omission, omitted, operation, oral, ordinal, ordinate, origin.

Pantograph, parabola, parallel, parenthesis, partial, particularly, partition, pentagon, percentage, perimeter, perpendicular, plotting, point, polygon, polynomial, positive, practical, preceding, precise, preferred, prefix, prime, principal, principle, prism, product, profile, proportion, protractor, pyramid, Pythagoras.

Quadrant, quadratic, quadrilateral, quantity, quart, quarter, quotient.

Radical, radicand, radius, rate, ratio, rational, rationalize, ratios, real, reasonable, reciprocal, rectangle, rectangular, reflex, relative, remainder, representation, respectively, reverse, rod, root.

Satisfy, scale, secondary, sector, segment, separate, series, sextant, significant, similar, simplify, simultaneous, solid, solution, spiral, sphere, square, standard, statement, statistical, straight, subtrahend, subtraction, substitution, succession, sufficient, sum, supplementary, surd, surface, symbol.

Tabulation, tangent, technical, temperature, term, theorem, theory, therefore, thermometer, thousandth, transformation,

translation, transpose, trapezoid, triangle, trinomial, trisect, truncated, typical.

Uniform, unit, unity.

Variable, variation, velocity, verbal, verification, verify, vertex, vertical, vinculum, volume.

Zero, zone.

## XII. BIBLIOGRAPHY.

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### *Practical Mathematics.*

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### *Standards.*

First Year Algebra Scales. Henry G. Hotz. Teachers College.

Mathematics. Rugg and Clark. University of Chicago.

*Note.*—For additional bibliography consult the MATHEMATICS TEACHER for September, 1915, and volumes of Smith and Young. In the *School Review* for September, 1917, pages 520-526, E. R. Breslich reviews "Recent Literature on Secondary School Mathematics."

There are valuable works by two Englishmen, George St. Lawrence Corson and T. Percy Nunn.

## XIII. NOTE ON A COLLEGE PREPARATORY COURSE IN INTERMEDIATE ALGEBRA.

The following is an outline of the chief additional topics in an intermediate college preparatory course to follow the first-

year course given in this syllabus. Such a skeleton outline may be helpful to many teachers, and it may serve to define our intended scheme for first-year work by showing what is left to be done in order to meet the present college requirements. Such a course needs a full year of work, as is implied in the College Entrance Examination Board recommendation of two units credit for algebra. Much of the time would be used on review topics, with problems of a difficulty more appropriate to the added maturity of the pupils and to the less remote contingency of college entrance.

1. *Factoring*.—(a) Difference of two squares, particularly the trinomial type,  $a^4 + a^2b^2 + b^4$ .

(b) Factor theorem.

(c) Binomial  $x^n + y^n$  studied with the aid of the factor theorem, particularly  $x^3 + y^3$ .

(d) H. C. F. and L. C. M. by factoring, not neglecting factors that differ only in sign, e.g.,  $a - b$  and  $b - a$ .

2. *Fraction*.—(a) Addition and subtraction including facility with such difficulties of signs as that mentioned in the preceding topic.

(b) Complex fractions.

3. *Exponents and Radicals*.—(a) Interpretation and manipulation of expressions involving negative, fractional, and zero exponents.

(b) Changing surds to the same index.

(c) Operations with surds of any order.

(d) Rationalization of a binomial surd denominator of the second order.

(e) Square root of a binomial quadratic surd.

(f) Radical equations.

4. *Quadratic Equations*.—(a) Completing the square more thoroughly mastered, including the derivation of the formula.

(b) Theory of quadratic equations; the relations between roots and coefficients, and the nature of the roots.

(c) Equations in quadratic form.

(d) Simultaneous quadratics, including homogeneous equations and those solvable by dividing one equation by the other.

5. *Binomial Theorem* for positive integral exponents including the formula for the  $n$ th term.

6. *Arithmetical and Geometric Progression.*

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